

Formulas for gear calculation – external gears

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The **involute gear** profile is the most commonly used system for gearing today. In an involute gear, the profiles of the teeth are *involute of a circle*. (The involute of a circle is the spiraling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle.)

In involute gear design, all contact between two gears occurs in the same fixed, flat plane even as their teeth mesh in and out. Further, the contacting surfaces are always perpendicular to the plane of contact, so the dominant contact forces (in a well lubricated system) are always parallel to the plane. This way, the moment arms are kept constant. This is key to minimizing the torque/speed variations which produce vibration and noise in lower quality gears. Note that the involute profile does not prevent the teeth from scraping each other every time they mesh, and this is the dominant source of wear. It is not possible to design a gear tooth profile which rolls through the mesh without friction.

The figure N°1 show the involute curve generation with the most important elements.

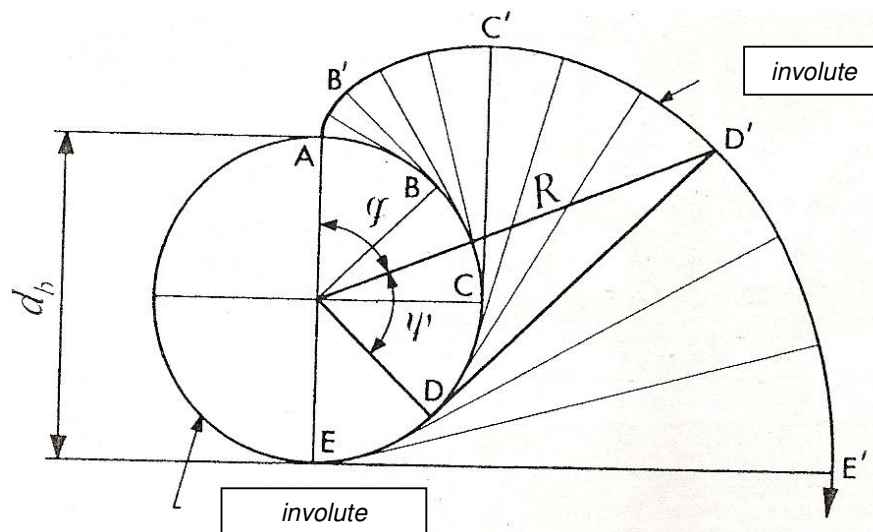


Fig. N°1

The parametric equations of involute of a circle are::

$$R = \frac{r_g}{\cos \psi} = r_g \cdot \sec \psi$$

$$\varphi = \tan \psi - \psi = \text{inv}(\psi)$$

Because we must use many formulas, it's better to show the list of symbols and indices utilized.

<i>Meaning of symbols</i>			
a	Center distance	m	Module
α	Pressure angle	Q	Dimension over pins or balls
β	Helix angle	r	Radius
d	Diameter	R_a	Radius to start of active profile
g	Length of contact	s	Tooth thickness on diameter d
g_1	Legth of recession	\bar{s}_{os}	Chordal thickness
g_2	Length of approach	t	Pitch
h_f	Dedendum	w	Chordal thickness over z' teeth (spur gears)
h_k	Addendum	W	Chordal thickness over z' teeth (helical gears)
h_0	Corrected addendum	z	Number of teeth
h_r	Whole depth	x	Profile correction factor
l	Tooth space		
<i>Meaning of indices</i>			
b	Reffered to rolling diameter	n	Reffered to normal section
c	Reffered to roll diameter of basic rack	o	Reffered to pitch diameter
f	Reffered to root diameter	q	Reffered to the diameter throug balls center
g	Reffered to base diameter	r	Reffered to balls
k	Reffered to outside diameter	s	Reffered to transverse section
i	Reffered to equivalent	w	Reffered to tool

Determination of base tooth thickness from a known thickness and vice-versa.

$$s_g = (\psi + 2\vartheta) \cdot r_g \quad \text{where } \psi = \frac{s}{r}$$

and vice-versa is:

$$s = \left(\frac{s_g}{r_g} - 2 \cdot \text{inv} \alpha \right) \cdot r$$

Where we can see $\vartheta = \text{inv} \alpha$ where α is the pressure angle in the points E and D. We can understand immediately the importance of the function $\text{inv} \alpha$.

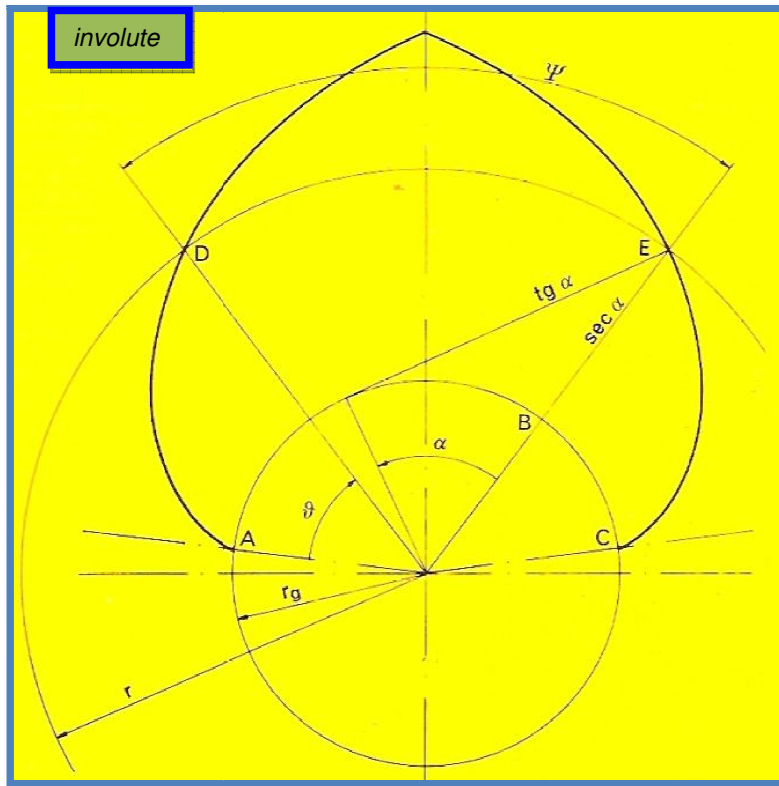


Fig. N°2

Cylindrical spur gears with standard profile

On base to figure N°3 the following relation are valid:

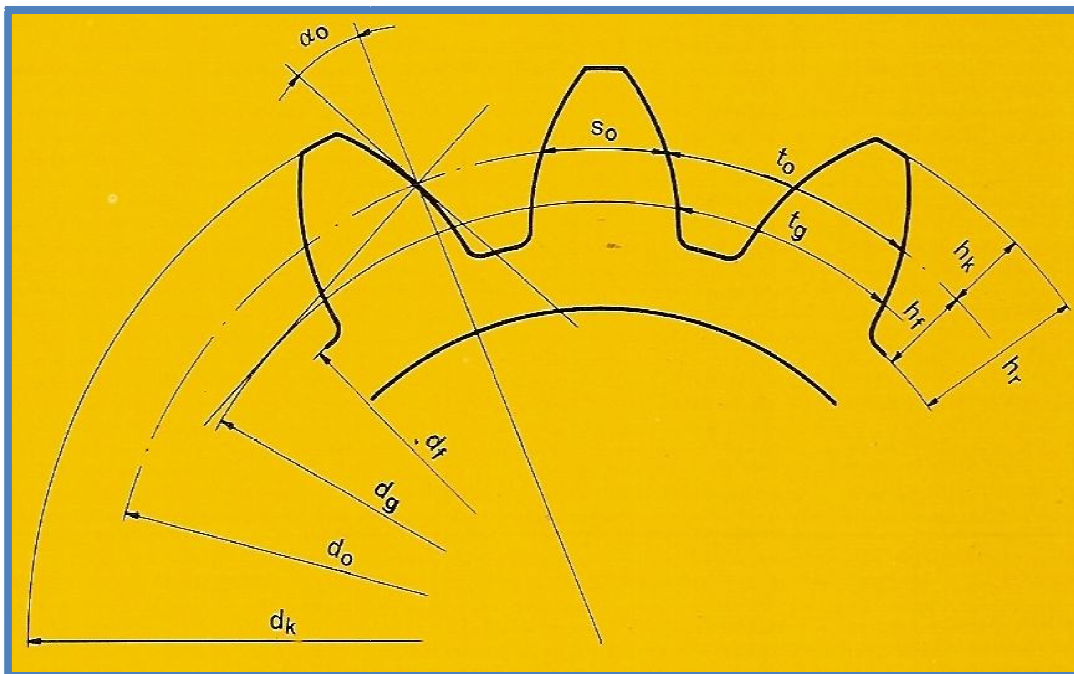
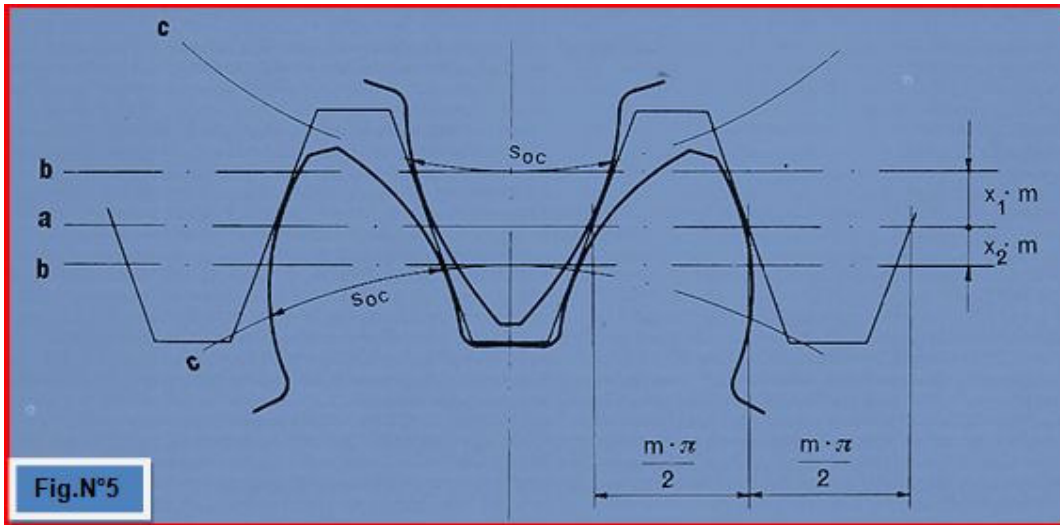


Fig. N°3

$$d_k = m(z + 2 + 2x - 2k)$$

$$h_r = m(2,25 - k) \quad \text{or} \quad h_r = m(2,167 - k)$$



Cylindrical helical gears with standard profile (Fig. N°6)

$$d_o = m_s \cdot z$$

$$d_g = d_o \cdot \cos \alpha_{os}$$

$$m_n = m_s \cdot \cos \beta_o$$

$$t_{on} = m_n \cdot \pi$$

$$h_k = m_n$$

$$h_r = h_k + h_f$$

$$d_f = d_o + 2 \cdot h_f$$

$$t_{os} = m_s \cdot \pi$$

$$t_{gs} = t_{os} \cdot \cos \alpha_{os}$$

$$\tan \alpha_{on} = \tan \alpha_{os} \cdot \cos \beta_o$$

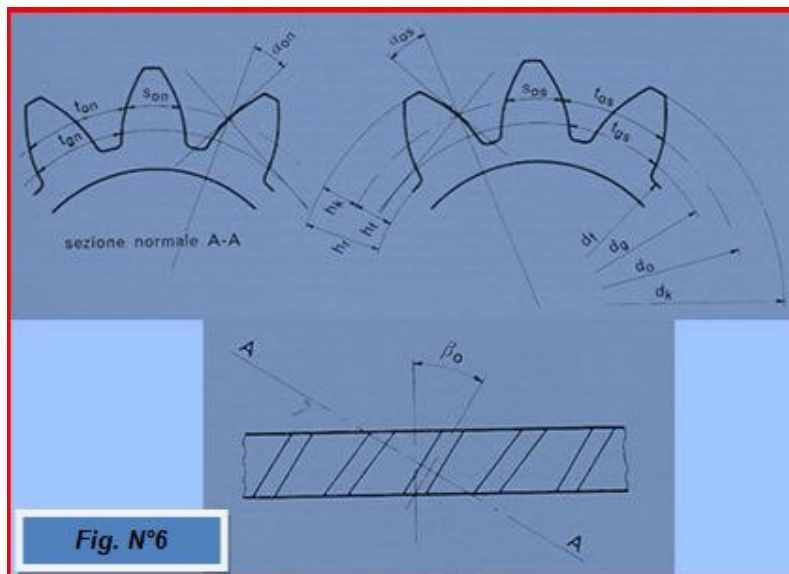
$$t_{gn} = t_{on} \cdot \cos \alpha_{on}$$

$$h_f = \frac{7}{6} \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n$$

$$d_k = d_o + 2h_k$$

$$S_{on} = \frac{\pi \cdot m_n}{2}$$

$$S_{os} = \frac{\pi \cdot m_s}{2}$$



Cylindrical helical gears with corrected profile

a)- Without center distance variation (fig.N⁴).

$$h_k = m_n + x \cdot m_n$$

$$h_f = \frac{7}{6} \cdot m_n - x \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n - x \cdot m_n$$

$$s_{onc} = \frac{m_n \cdot \pi}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_{on}$$

$$s_{osc} = \frac{m_s \cdot \pi}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_{os}$$

b)- With center distance variation (fig. N⁵)

$$\text{inv } \alpha_{bs} = \text{inv } \alpha_{os} + 2 \tan \alpha_{on} \frac{x_1 + x_2}{z_1 + z_2}$$

$$a_b = a \frac{\cos \alpha_{os}}{\cos \alpha_{bs}} \quad d_b = \frac{d_g}{\cos \alpha_{bs}}$$

$$s_{bs} = r_b \left[\frac{s_{osc}}{r_o} + 2(\text{inv } \alpha_{os} - \text{inv } \alpha_{bs}) \right]$$

$$k = \frac{z_1 + z_2}{2} \left[2 \frac{x_1 + x_2}{z_1 + z_2} - \frac{1}{\cos \beta_o} \left(\frac{\cos \alpha_{os}}{\cos \alpha_{bs}} - 1 \right) \right]$$

$$d_k = m_n \left(\frac{z}{\cos \beta_o} + 2 + 2x - 2k \right)$$

$$h_r = m_n(2,25 - k) \quad \text{or} \quad h_r = m_n(2,167 - k)$$

Length of contact and contact radius R_a

$$\rho_{k1} = \sqrt{r_{k1}^2 - r_{g1}^2} \quad \rho_{k2} = \sqrt{r_{k2}^2 - r_{g2}^2}$$

$$g = \rho_{k1} + \rho_{k2} - a_b \cdot \sin \alpha_b$$

$$g_1 = \rho_{k1} - r_{b1} \cdot \sin \alpha_b$$

$$g_2 = \rho_{k2} - r_{b2} \cdot \sin \alpha_b$$

$$R_{a1o1} = \sqrt{(\rho_{k1o1} - g)^2 + r_{g1o2}^2}$$

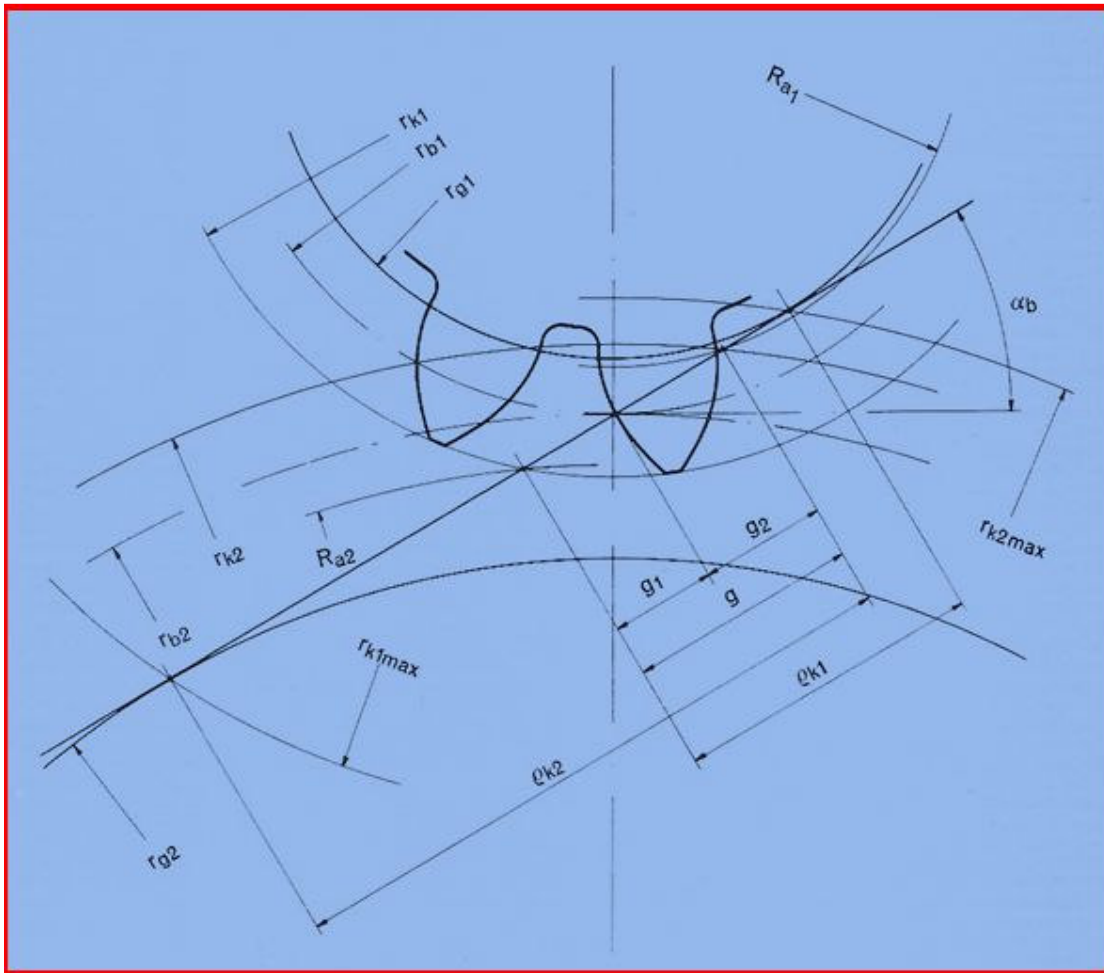


Fig. N°9

Interference

Maximum outside diameter without interference:

$$d_{kmax} = \sqrt{d_g^2 + (2a_b \cdot \sin \alpha_b)^2}$$

Chordal thickness and corrected addendum (fig.10)

$$\frac{s_o}{\pi \cdot d} = \frac{\delta}{2 \cdot \pi} \quad \text{and} \quad \delta = \frac{2 \cdot s_o}{d}$$

$$\bar{s}_o = d \cdot \sin \frac{\delta}{2}$$

$$h_o = h_k + \frac{d}{2} \left(1 - \cos \frac{\delta}{2} \right)$$

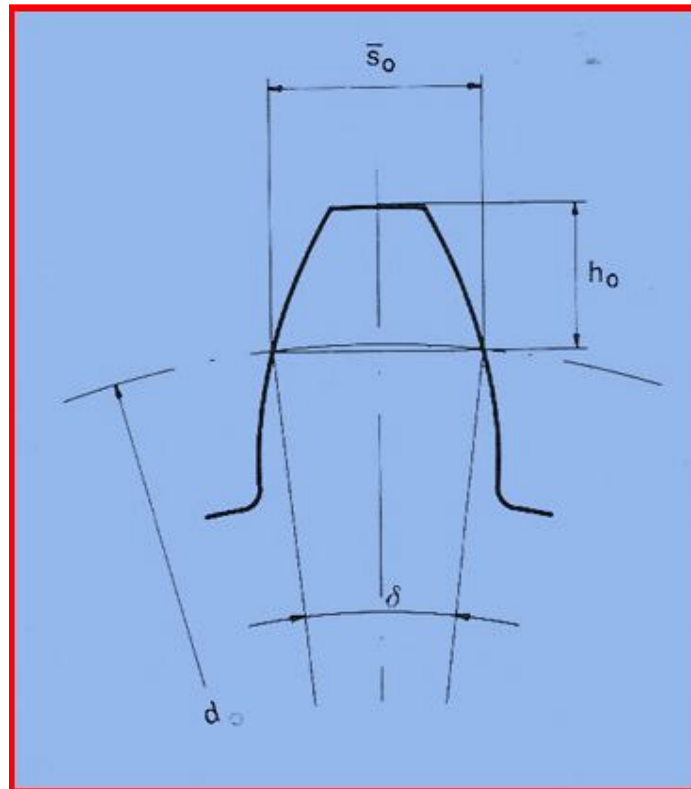


Fig. N°10

Span measurement over Z' teeth

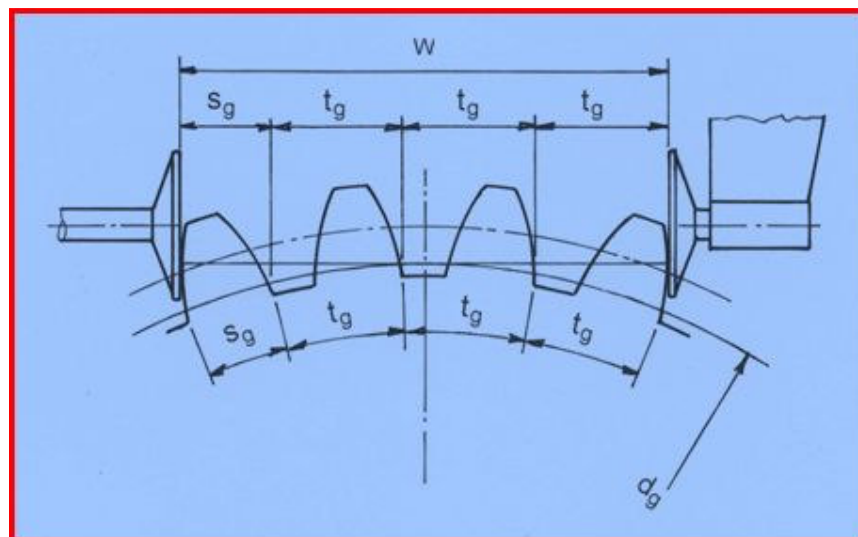


Fig. N°11

For spur gears:

$$w = m \cdot \cos \alpha_o \cdot \left[(z' - 1)\pi + \frac{s_o}{m} + z \cdot \text{inv } \alpha_o \right] + 2 \cdot x \cdot m \cdot \sin \alpha_o$$

The number of teeth Z in the span is:

$$z' = z \frac{\alpha_o}{180^\circ} + 0,5 \quad (\text{with } \alpha_o \text{ in degree})$$

The result should be rounded off to the next highest whole number.

For gear with profile correction $\geq 0,4 \cdot m$ the formula is:

$$z' = z \frac{\alpha_o}{180^\circ} + 0,5 + z \cdot \frac{\tan \alpha_x}{\pi} - (z + 2x) \frac{\tan \alpha_o}{\pi} \quad \text{where}$$

$$\tan \alpha_x = \sqrt{\tan^2 \alpha_o + 4 \frac{x}{z} \sec^2 \alpha_o + 4 \left(\frac{x}{z}\right)^2 \sec^2 \alpha_o}$$

For helical gears:

$$w = m_n \cdot \cos \alpha_{on} \cdot \left[(z' - 1)\pi + \frac{s_{on}}{m_n} + z \cdot \text{inv } \alpha_{os} \right] + 2 \cdot x \cdot m_n \cdot \sin \alpha_{on}$$

where

$$z' = z \left(\frac{\alpha_{os}}{180^\circ} + \frac{\tan \alpha_{os} \cdot \tan^2 \beta_g}{\pi} \right) + 0,5 \quad (\text{con } \alpha_o \text{ in degree})$$

This method of checking can only be used when

$$b \geq w \cdot \sin \beta_g + b_M \cdot \cos \beta_g$$

where β_g is the length of the contact lines between little dishes and teeth (figura N°12).

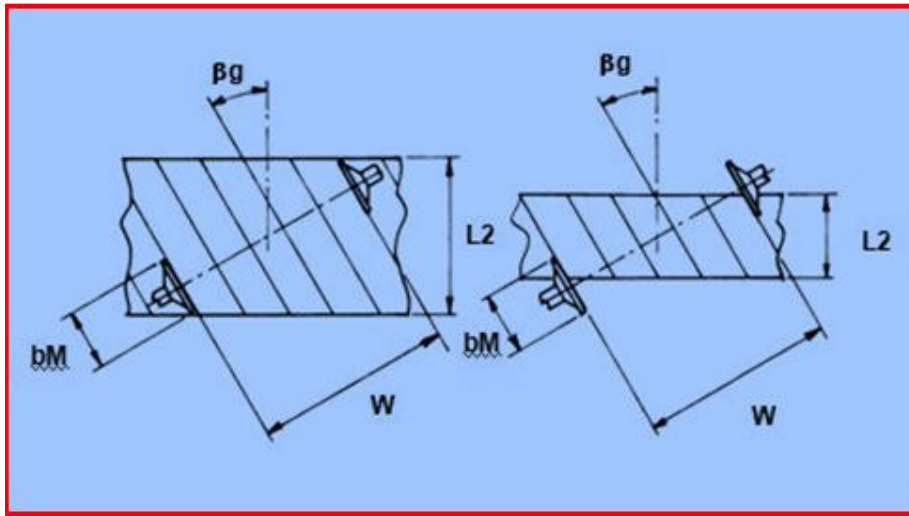


Fig.N°12

If the gear has a large profile correction, then z should be calculated as follow:

$$z' = z \frac{\alpha_{os}}{\pi} + 0,5 + \frac{z}{\pi} \cdot \frac{\sqrt{\tan^2 \alpha_{on} + 4 \frac{x}{z} \cos \beta_o \left(1 + \frac{x}{z} \cos \beta_o \right) (\tan^2 \alpha_{on} + \cos^2 \beta_o)}}{\cos \beta_o (\sin^2 \alpha_{on} + \cos^2 \beta_o \cdot \cos^2 \alpha_{on})} - \frac{z}{\pi} \tan \alpha_{os} - 2 \frac{x}{\pi} \tan \alpha_{on}$$

Dimension over pins or balls

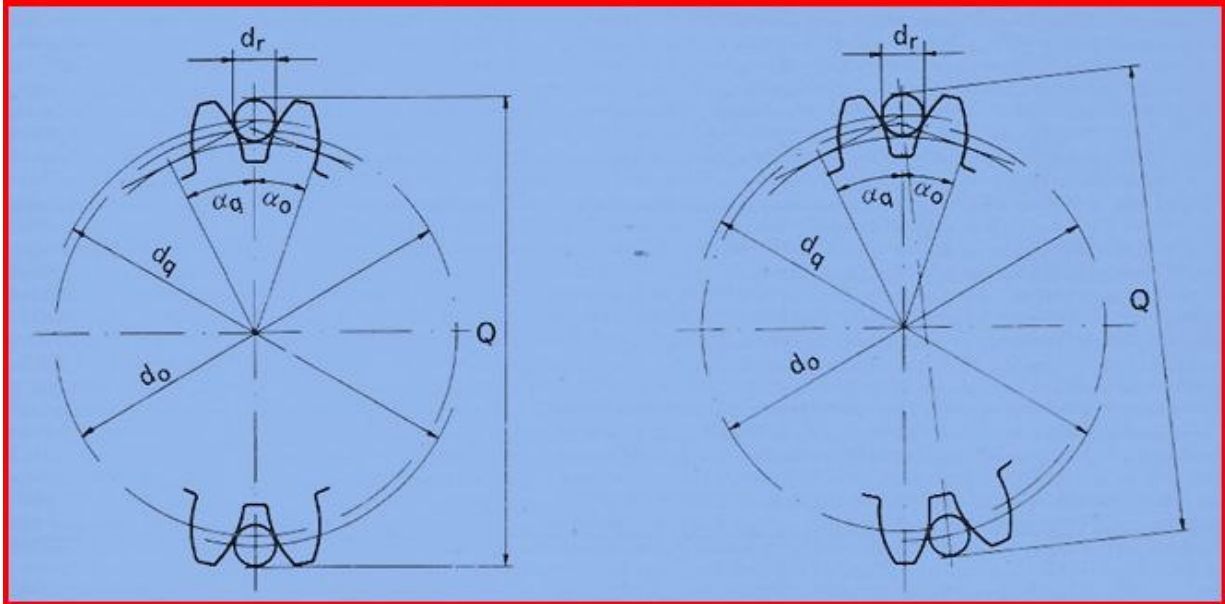


Fig.N°13

Spur gears with even number of teeth

$$\text{inv } \alpha_q = \text{inv } \alpha_o + \frac{d_r}{2r_o \cdot \cos \alpha_o} - \frac{l_o}{2r_o} \quad \text{from which we have } \alpha_q$$

$$r_q = r_o \frac{\cos \alpha_o}{\cos \alpha_q} \quad Q = 2 \cdot r_q + d_r$$

Spur gears with odd number of teeth

With the same α_q e r_q we obtain $Q = 2 \cdot r_q \cdot \cos \frac{\pi}{2z} + d_r$

Helical gear with even number of teeth

$$\text{inv } \alpha_{qs} = \text{inv } \alpha_{os} + \frac{d_r}{2r_{os} \cdot \cos \beta_o \cos \alpha_{on}} - \frac{l_{os}}{2r_{os}}$$

$$r_{qs} = r_{os} \frac{\cos \alpha_{os}}{\cos \alpha_{qs}} \quad Q = 2 \cdot r_{qs} + d_r$$

Helica gears with odd number of teeth

With the same α_{qs} e r_{qs} we obtain $Q = 2 \cdot r_{qs} \cdot \cos \frac{\pi}{2z} + d_r$